THC White Paper Series #3
Low Interest Rate Regime: Challenges and Solutions

MANAGE OPTION RISK

LOCAL VOLATILITIES MODEL

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PROLOGUE

The low rate regime has vast ramifications on balance sheet and portfolio performance. Recently, European global banks such as Deutsche Bank, HSBC, and others have refocused their business models in response to the low-interest rates. Such re-organizations have heightened awareness of the urgency to take proactive business actions in this environment.

I began this series of papers by urging bankers to sharpen their measures of risk and value when margins are tight. For example, to date, many banks have not updated their interest rate models since the early 1990s to reflect the low-interest rates in 2019. Can these models still measure interest rate risk accurately? Interest rate models affect credit and liquidity risks as well, and in turn, their impacts permeate to broader measures of a bank's performance.

Paper 1, "Arbitrage-Free Interest Rate Models," describes an interest rate model, the Local Volatilities Model, and explains the importance of re-examining the measure of loan profitability, accounting for the funding cost. Paper 2 provides a methodology to measure profitability on a risk-adjusted basis, and addresses the issues in financial modeling.

This paper will continue the discussion extending to describe strategies to manage option risks on the balance sheet.

INTRODUCTION

Option, option everywhere - Option Risk everywhere. The current low-interest rates have significant repercussions on capital markets. For example, many banks experienced a torrent of residential mortgage prepayments and called bonds during the third quarter of 2019, lowering financial income significantly. Regulators call such prepayment spikes "option risk." The Asset-Liability Committee (ALCO) has to manage option risk, especially when there are many options embedded on the balance sheet.

Residential mortgage prepayment, caps/floors on ARM products, callable/puttable structured Advances, customer withdrawals, bond call The interest rate has dropped from 4% to 2%. The current low-interest rates have significant provisions, and many other contingencies are options on the balance sheet. They are options because their projected cashflows can vary greatly as triggered by changing market conditions.

For example, residential mortgage prepayments rise sharply when the interest rate falls below the mortgage rate at a certain level. Adjustable-rate mortgages become fixed-rate mortgages when the index rate reaches the caps or floors; issuers call bonds when a callable bond value reaches a certain level. There are many contingencies on balance sheet instruments that expose performance to option risks.

Managing balance sheet performance with options cannot be based on a single scenario, as reported in the financial statements. Instead, multiple probability-weighted scenarios are required to access the sharp changes in the cash flow of options. Managers have to anticipate option impact on performance to avoid potential loss and enhance returns.

TERM STRUCTURE OF VOLATILITIES

Local Volatilities Model is represented by the Product of Term Structure of Volatilities and the Rate Distributions

LOCAL VOLATILITIES MODEL

Paper 1 has discussed that the interest rate model captures multiple market risk drivers. In addition to the shape of the yield curve, there are five risk drivers of the volatility surface.

The model specification is relatively simple and robust. There are only five parameters to specify the model. (i) Parameter [a] specifies the short term volatility. (ii) Parameter [b] specifies the short-term increase or decrease of volatility. (iii) Parameter [c] specifies the decay of the volatilities over time; (iv) [d] is the long term volatilities. (v) Parameter [p] is the skewness of the binomial distribution.

Recall, Paper 1 derives that the term structure of volatility surface with independent parameters (n, i) is given by where the volatilities are measured in basis points, as opposed to proportional to the interest rate level. The state i is related to the rate level, in the sense that i is the number of upward shifts of the rate.

The Local Volatilities Model can be specified as

$\Delta r\,(n,I) = \Phi(n,i) + \sigma(i\,,n)\,\Delta z$
Where
$\Delta r\,(n,I)$ is the change in rate at time n and state I
$\Phi(n,i)$ is the term adjusted to ensure rates are arbitrage-free and is derived in Paper 1
Δz is the random walk up or down
$\sigma(i\,,n) = f(n).n.g(i)$ is the volatility
where by $f() = (a+b*n)*\exp(-c*n)+d$ is the term structure of volatilities
And $g(i)$ is the rate distribution.

I will first discuss the term structure of volatilities.

TERM STRUCTURE OF VOLATILITIES

The Shape of the Term Structure of Volatilities inferred from the capital market pricing of uncertainties by the level and timing

HISTORICAL TRENDS

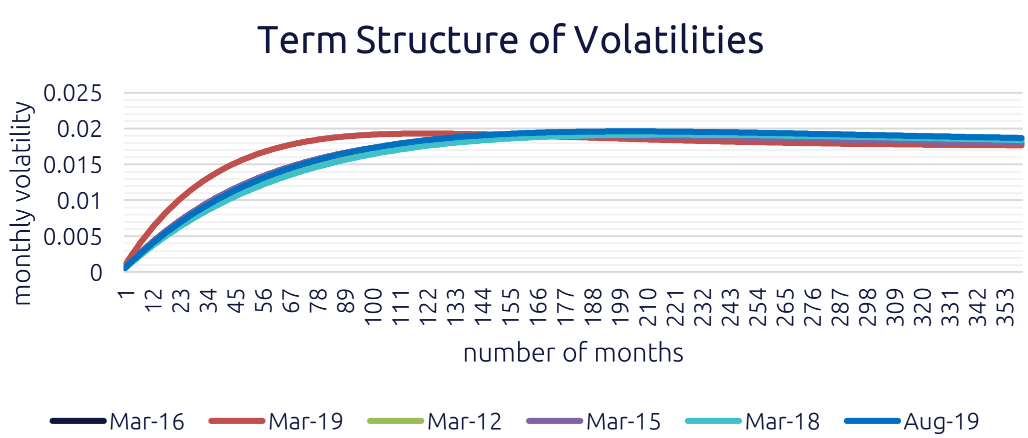
To underscore the transparency of the Local Vol model, I provided a historical trend of the term structure of volatilities. The graph depicts the projected one-month interest rate volatilities of the Local Volatilities Model. I chose the period 3/2016 to 8/2019 to depict the term structure of volatilities implied by the swaptions.

The shape of the term structure of volatilities historically can take on a shape of decay or humped representing the market pricing of uncertainties.

Results show that the term structure of volatilities change as the market anticipates different level of uncertainties.

The table presents the calibrated parameters of the Local Vol Model.

	a	b	c	d	p
	sht vol	chg sht vol	vol decay	lg vol	skewness
Mar-16	-0.0169	0.0002	0.0113	0.0172	0.5518
Mar-19	-0.0171	0.0003	0.0176	0.0175	0.5710
Mar-12	-0.0169	0.0002	0.0100	0.0172	0.5508
Mar-15	-0.0169	0.0002	0.0113	0.0172	0.5513
Mar-18	-0.0170	0.0001	0.0096	0.0172	0.5585
Aug-19	-0.0175	0.0002	0.0104	0.0177	0.5540



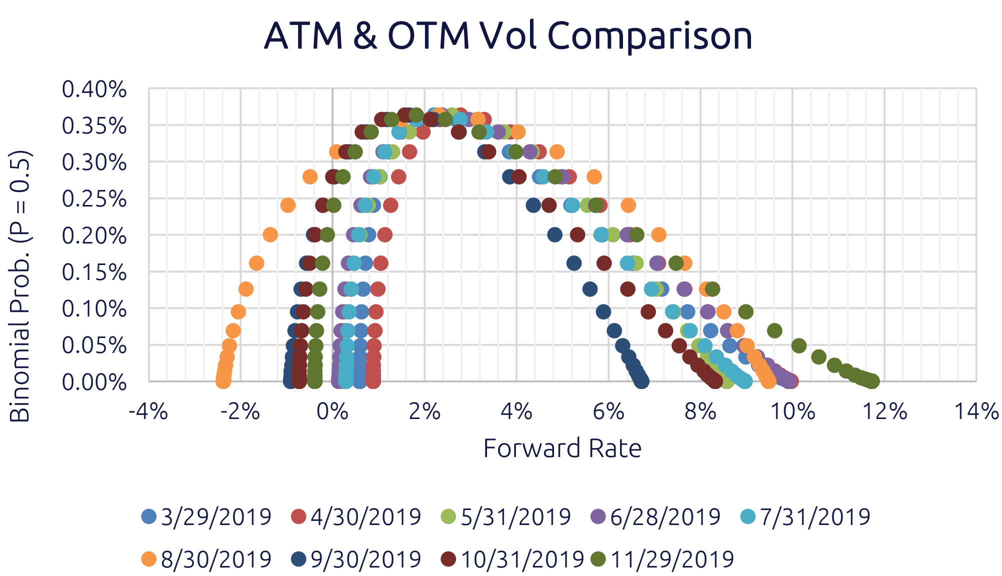
RATE DISTRIBUTION

The Rate Distribution presents the capital market relative pricing of rate rising vs falling.

I provide a historical trend of the Probability Distributions of Rates. Rate Distribution depict the projected one-month interest rates specified by the Local Volatilities Model.

I chose the period 3/2019 to 11/2019 to depict the period Rate Distribution as implied by the At-the-Money and Out-of-the-Money swaptions. I present the changing market views, as inferred by in-the-money and out-of-the-money swaption prices.

The results show that some projected implied rates have been significantly negative since August 2019. Figure 1 is a scattered pot of one month swap rate at a 10 year time (n= 120) against the probability of reach the state i. Specifically scattered points are [r(n, i), b(n,i)] for n =120 and I = 0, 1,2, …, 120
b(n, i) is the symmetric binomial distribution.



OTM IMPACT ON RATE DISTRIBUTION

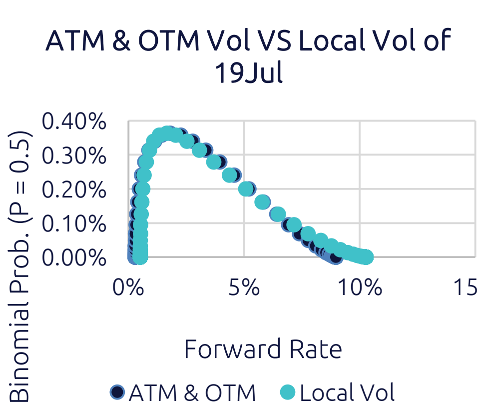
Out-of-the-Money options have significant impact on the Rate Distribution

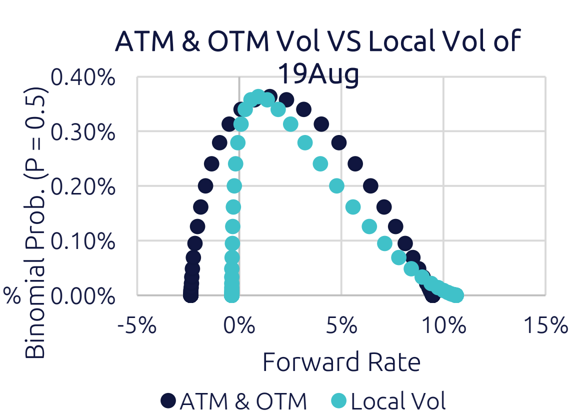
Many interest rate models are calibrated with only At-the- Money Options. But the results show below that when interest rates are low, the Out-of-the-Money options can have significant impact on the Rate Distribution, and hence the pricing of caps and floors.

The right hand figure compares the Rate Distributions calibrated by OTM and ATM swaptions vs that calibrated by ATM swaptions only. The impact of OTM on the distribution is clearly presented by the chart on the right hand side.

By way of contrast, when interest rates were higher in July, the impact of the OTM on the rate distribution was much less significant.

The results show that the implied distribution of one-month rates can be negative. August 2019, the rates can fall below -2%. The implication is that by using Out-of-the-Money options in calibrating an interest rate model, traders can identify the pricing of caps and floor accurately because the distribution of rates become an important determinant of these embedded option value.





IMPLICATIONS; SKEWNESS AND OTM

Traders can more accurately identify the pricing of caps and floor and the impact of stress tests

This paper shows that managers can manage option risk as follows.

- Offer Adjustable Rate Mortgages (ARMs) with optimally designed floor rates or cap rates that can be profitable because there are divergent views of the future rate levels. An ARM floor of 2.5% is valuable to lenders but may be acceptable to the customers. Likewise, a 5% cap does not affect the loan value much but can offer customers significant interest rate risk protection.
- Design caps and floors as schedules to optimize loan performances since the market perceives the rate distributions are slightly asymmetrical as below.
- Identify the effect of mean reversion of the rate surface shape as the five movements drive the volatilities; global events may increase short term volatilities while inflation concern may affect the long-term volatility; the "media effect" may affect the rate surface temporarily.
- Bifurcate 1-4 family residential loans into option and cashflows so that the options can be managed while the cash flows can be swapped, as discussed in Part 2.

Managing fixed-income options is analogous to managing stock options, albeit that there are more risk drivers for the former type of options.

CONCLUSIONS

This paper presents three main results:

- *The term structure of volatilities identifies the market perceived uncertainties of short term and long term events;*
- *Rate Distributions identify the two side tail distributions of rates;*
- *Calibration should use both OTM and ATM options to price caps/floors structures correctly embedded in loans.*

Local Volatilities determine the values and risks of options. Option value increases with volatilities. However, different options are affected by different parts of the Rate Distribution and Terms. Some option value is affected by long term volatilities, such as 30-year residential fixed-rate mortgages while others are affected in the low rate areas, such as floors.

Similarly, the caps and floors of adjustable-rate mortgages are affected by the Rate Distribution. Therefore, the Rate Distribution and Term Structure of Volatilities can provide analytics for hedging, total return strategies, and yield enhancement lending.

Being able to manage option risk can protect potentially significant losses. At the same time, applying dynamic hedging as in arbitrage-free models such as Black-Scholes and Ho-Lee methodology can create synthetic options and offer profit opportunities.

Options are everywhere, and their impact on a balance sheet can be sudden and significant, often described as “falling off the cliff” or “convexity.” Option risk is real, as evident by the current residual mortgage prepayment spike, resulting in many banks reporting realized losses.

Option risk should be measured, monitored, and managed. The Rate Distribution and Term Structure of Volatilities provide economical performance indicators.

EPILOGUE

This paper has introduced the Local Volatilities and describes the benefits of using a robust interest rate model. Paper 4 will continue this discussion, using the model to manage balance sheet items and demonstrate the peril of using legacy models developed in the 1990s.

TECHNICAL NOTES

FROM BLACK-SCHOLES TO HO-LEE

Managers should begin with a financial model of an option to identify the option value and risk. Most capital market participants are familiar with the Black-Scholes model (1974) in pricing stock options. The model asserts that options can be "manufactured" by buying the underlying stock with cash, continually adjusting the ratio between the stock and cash. That is: algorithmic trading can replicate options, creating synthetic options.

Ten years later, Ho-Lee (1984) extends the Black-Scholes model to fixed-income instrument options. Unlike stock options, fixed-income options do not depend only on the stock price and its volatility. Fixed-income options depend on (1) the shape of the yield curve, for example, the swap curve and (2) volatilities of the rates along the yield curve, often called the Vol Surface. The Vol Surface, in turn, is specified as the time horizon and asymmetrical likelihood of rate rising as opposed to rate falling. Therefore, the behavior of a fixed-income option depends on many market factors, making these options difficult to visualize and analyze. To date, many banks rely on black-box interest rate models without the understanding of their projected interest rates.

The Local Volatilities Model can be explained from the Heath-Jarrow-Morton Model (HJM) framework. The HJM model re-formulate the Ho-Lee model in terms of the forward rates. More precisely the HJM model is represented by the equation

$$df = c(t, f)dt + \sigma (t, f)\, dz$$

Where $c(t, f)$ is the convexity adjustment, derived from the arbitrage-free modeling condition. $\sigma (t, f)$ is the rate volatilities. That is: the HJM model shows that options can be priced by using the discount curve and the rate volatilities only.

The Local Volatilities Model specifies the rate volatilities $\sigma (t, f)$ as a product of term structure of volatilities and rate distributions to ensure the model is transparent and robust.

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